Transonic Shock Interaction with a Tangentially Injected Turbulent Boundary Layer

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Described is a nonasymptotic triple-deck theory of transonic shock/turbulent boundary-layer interaction that takes into account the influence of upstream tangential injection on a curved wall. In addition to the Reynolds number and shock strength, the theory is parameterized by arbitrary values of the incoming boundary-layer shape factor and the wall jet maximum velocity ratio and its nondimensional height. Results of a comprehensive parametric study are then presented. It is shown that the wall jet effects significantly reduce both the streamwise scale and displacement thickening of the interaction zone. While increasing the upstream and downstream skin-friction levels, these effects also reduce the minimum interactive skin-friction coefficient and thus hasten the onset of local incipient separation at the shock foot.

Nomenclature

C_f	= skin-friction coefficient, $2\tau_w/\rho_{e_o}U_{e_o}^2$
ΔC_f	= skin-friction increment due to wall jet
C_p	= pressure coefficient, $2p'/\rho_{e_o}U_{e_o}^2$
C_x	= lateral spreading factor of wall jet
H	= boundary-layer shape factor, δ^*/θ^*
H_i	= incompressible shape factor
K_B	= curvature of wall in interaction region
ℓ_u,ℓ_D	= upstream and downstream influence distances
L	= distance to shock location
M	= Mach number
P	= static pressure
P'	= interactive pressure perturbation, $p-p_I$
Δp	= pressure jump across incident shock
Re_L,Re_δ	= Reynolds numbers based on length L and boundary-layer thickness, respectively
S_w	= nondimensional wall shear function of wall jet
T	= absolute temperature
u',v'	= streamwise and normal interactive disturbance velocity components, respectively
ΔU	= wall jet component of total velocity profile
U_o	= undisturbed incoming boundary-layer velocity in x direction
x,y	= streamwise and normal distance coordinates (origin at the inviscid shock intersection with the wall)
$y_{\max}$	= location of $U_{\max}$
β	= $\sqrt{M_I^2-1}$
γ	= specific heat ratio
δ	= boundary-layer thickness
δ^*	= boundary-layer displacement thickness
$\delta_{\rm mix}$	= wall jet mixing thickness
$\delta_{\rm SL}$	= inner deck sublayer thickness
ϵ_T	= kinematic turbulent eddy viscosity
ϵ_T'	= interactive perturbation of turbulent eddy viscosity
η	= y/δ_o
μ	= ordinary coefficient of viscosity
ν	= μ/ρ
ω	= viscosity-temperature dependence exponent, $\mu\sim T^\omega$
ρ	= density
θ^*	= boundary-layer momentum thickness
τ	= total shear stress
τ'	= interactive perturbation of total shear stress

Subscripts

i	= undisturbed inviscid values ahead of incident shock
e	= conditions at the boundary-layer edge
inc	= incompressible value
inv	= inviscid disturbance solution value
max	= velocity profile maximum due to wall jet
o	= undisturbed incoming boundary-layer properties
w	= conditions at wall (adiabatic)

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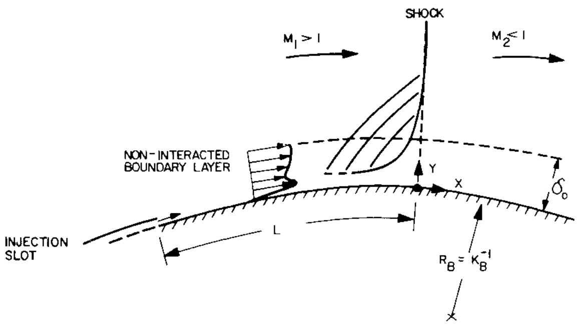
Introduction

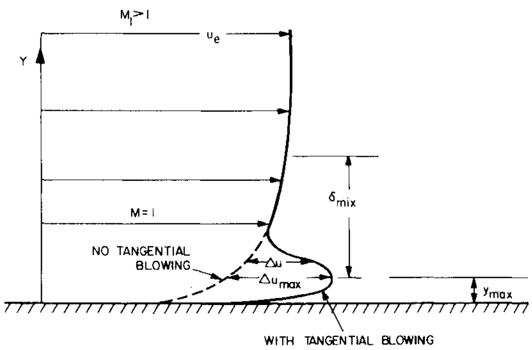
THE influence of tangential slot injection on turbulent boundary-layer behavior has been extensively studied in low-speed flows on circulation-controlled airfoils and slotted flaps, in film cooling applications, and for separation control in inlets and diffusers. In recent years, applications of such injection have also arisen in supercritical transonic flows containing a local shock wave; however, little is understood about how the resulting shock/boundary-layer interaction (SBLI) alters the influence of tangential injection. Conversely, it is of interest to know how SBLI effects are altered by the presence of upstream injection. This paper addresses these questions for steady, nonseparating two-dimensional turbulent boundary layers on adiabatic surfaces of small-to-moderate longitudinal curvature. The primary objectives were to develop a fundamental theory of a transonic SBLI region downstream of a tangentially injected turbulent boundary layer (Fig. 1) and then to carry out a parametric study of the relationship between the injection parameters and the SBLI zone physics.

Description of Theoretical Model

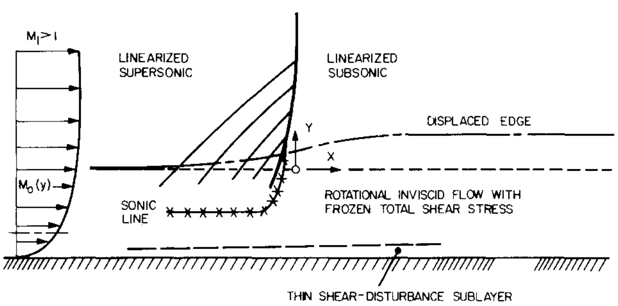
At high Reynolds numbers, the SBLI field has a basic triple-deck structure (Fig. 1) consisting of an outer potential inviscid flow astride the incident shock, a middle deck of frozen shear stress-rotational inviscid disturbance flow across most of the incoming boundary layer, and an inner shear-disturbance deck containing the interactive skin-friction perturbations (and possible incipient separation), plus most of the upstream influence. This disturbance field is analyzed by extending a fully turbulent nonasymptotic interaction theory[1] that has been extensively validated by experiment[2,3] and basic analysis.[4] The incoming turbulent boundary-layer profile is treated by an



a) Upstream injection slot.



b) Boundary-layer profile.



c) Triple-deck structure of interaction zone.

Fig. 1 Interaction zone with upstream tangential injection.

analytical composite law of the wall/wake model,[5] which in the absence of injection involves four arbitrary parameters[6]: preshock Mach number, displacement thickness Reynolds number, incompressible shape factor Hi_1, and the nondimensional wall curvature $K_B\delta_o$. The shape factor has a significant effect on the interactive properties observed in practice.[2]

The present problem entails a turbulent velocity profile $U_o(y)$ ahead of the SBLI zone that is altered by tangential injection. Sufficiently far downstream of the injection slot, experiments[7,8] have shown that it attains a fully developed "jet-bulged" shape (Fig. 1) composed of an unblown profile plus a wall jet component with a velocity maximum near the wall. This case is the one treated here, since regions upstream of the slot and far downstream where the profile maximum has disappeared can be handled by the existing "unblown" SBLI theory.[2] Besides altering the U_o and eddy viscosity profiles upon which the interaction depends, the upstream tangential injection may also alter the SLBI zone by influencing the outer inviscid flow (including shock wave location and shock shape) and by introducing new perturbation terms into the interactive disturbance equations. Regarding the inviscid flow aspect, it is noted that any injection effect on the unperturbed outer flow and shock location is automatically accounted for in whatever global inviscid solution code is used for the given body, since the present theory describes events *relative* to this location. The attendant shock obliquity due to the viscous displacement effect is accounted for accurately[10] by the approximation of sonic postshock flow corresponding to the effective lower normal Mach number $M_{1_{eff}} \simeq M_1 \sin\ (90° - 37.3\sqrt{M_1 - 1})$. Regarding possible new perturbation terms, a survey of experiments[9] shows that the law of the wall eddy viscosity model still applies below the jet maximum when the injection effect is moderate [$\Delta U_{max}/U_e \lesssim 1.0$, see Eq. (1)]. Since the thin inner deck of the SBLI region lies within this law of the wall region when there are no eddy viscosity-associated perturbation terms in the frozen turbulent flow of the overlying middle deck, the *form* of the usual triple-deck equations can be used here *provided* one fully accounts for the wall jet effects on the velocity profile shape (H_{i_1}), C_{f_o}, and Re_{δ^*}.

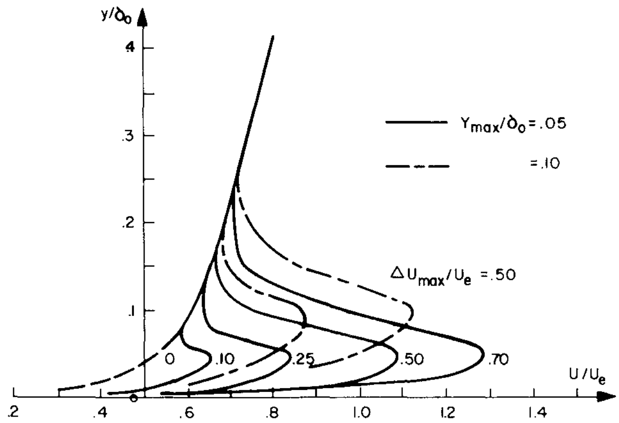


Fig. 2 Typical turbulent boundary-layer profiles with injection.

An appropriate analytical model of $U_o(y)$ was constructed as the sum of a wall jet component $\Delta U(y)$ and an "unblown" component based on Walz's model (Fig. 1),

$$U_o(y) = U_{\rm Walz}(y) + \Delta U(y) \tag{1}$$

where ΔU varies from $\Delta U(0) = 0$ (no slip) to the value ΔU_{max} at $y = y_{max}$ and then decays outwardly to zero, becoming negligible beyond some characteristic jet spread height δ_{mix} above y_{max}, where $\delta_{mix} + y_{max} << \delta$. *Above* y_{max}, following Carrierre et al.,[11] ΔU was represented by the modified sech^2 function

$$\frac{\Delta U}{u_e} = \frac{\Delta U_{max}}{u_e} \frac{{\rm sech}^2\{[(y - y_{max})/S_{mix}] + \Phi\}}{{\rm sech}^2\Phi}, \quad (y > y_{max}) \tag{2}$$

where $\Phi \equiv \frac{1}{2}[\ln(1 + C_x/2) - \ln(1 - C_x/2)]$ is a phasing factor insuring a maximum in total velocity at y_{max} and $C_x \equiv (\delta_{mix}/\Delta U_{max})[\partial\Delta u/\partial_y]_{max} \simeq 0.15$ is a lateral spreading constant. *Below* y_{max}, we require a functional representation of a reasonable montonic shape matching the value and slope of the upper $U(y)$ at y_{max}, while yielding a positive slope $S_w = y_{max}(\partial\Delta U/\partial y)_w/U_e$ and hence physically reasonable skin-friction increments of $\Delta C_f = S_w(\mu_w\delta/\mu_e Re_\delta)U_{max}/y_{max}$. A function that has proved satisfactory in this regard is

$$\frac{\Delta U}{U_e} = C_1 \frac{y}{y_{max}} - C_3\left[\exp\left(C_2 \frac{y}{y_{max}}\right) - 1\right], \quad (y > y_{max}) \tag{3a}$$

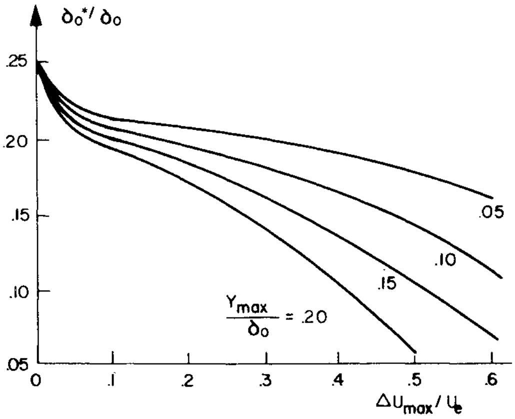
where the aforementioned conditions are fulfilled if the constants C_1, C_2, and C_3 satisfy

$$C_1(1 - \exp C_2) = U_{max}/U_e \tag{3b}$$

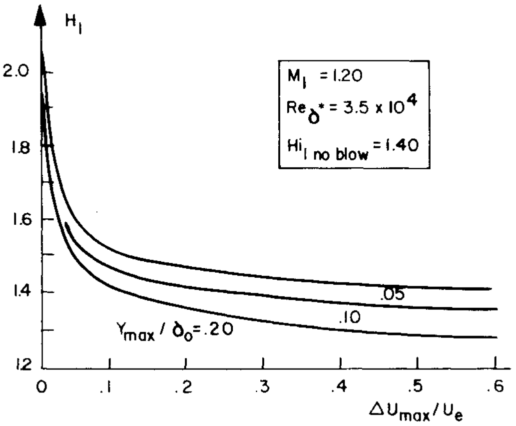
$$C_1 - C_3(\exp C_2 - 1) = y_{max}/\delta_o \tag{3c}$$

$$C_1 - C_3 = S_w \tag{3d}$$

This model captures the velocity overshoot and negative vorticity region features unique to this kind of flow (see Ref. 12 for a more detailed discussion), while enabling parametric sensitivity studies of how the injection influences the SBLI zone and providing for possible inclusion of later turbulent wall jet boundary-layer data. A typical set of the resulting profiles is illustrated in Fig. 2.

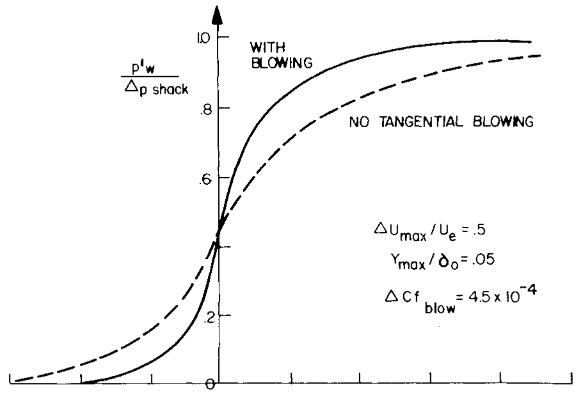


a) Displacement thickness.

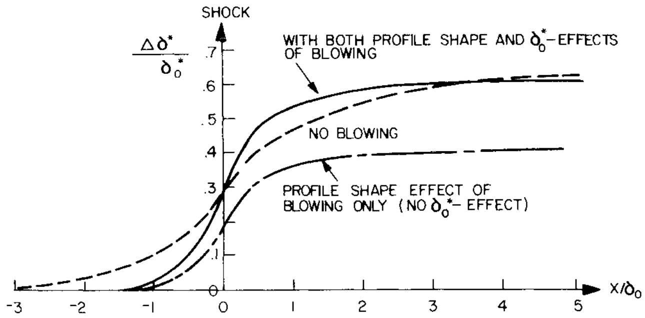


b) Shape factor.

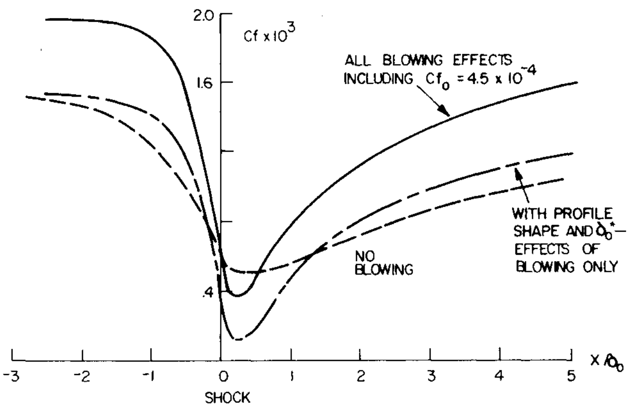
Fig. 3 Injection effect on integral properties of the incoming boundary layer.



a) Wall pressure.



b) Displacement thickness.



c) Skin friction.

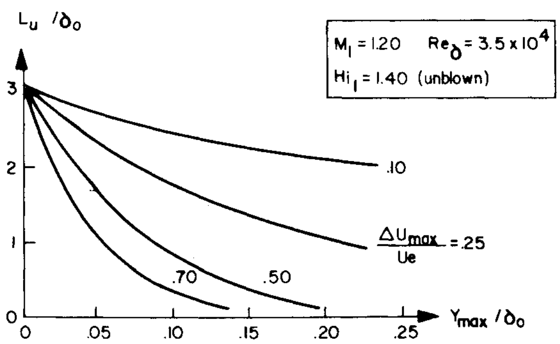
Fig. 4 Typical interaction zone solution properties.

Discussion of Results

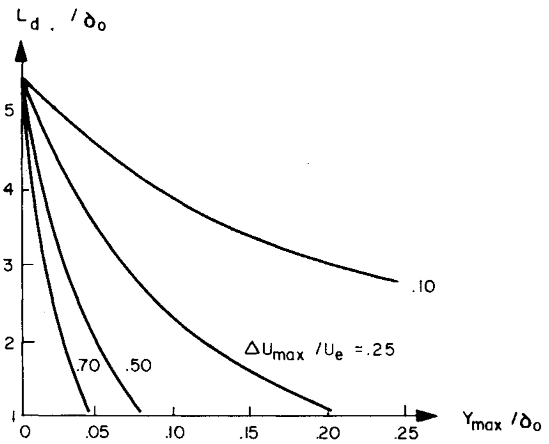
The tangential injection is represented by the two parameters, $\Delta U/U_e$ and $y_{\max}/\delta_o$, characterizing the magnitude and height, respectively, of the wall jet component effect; the auxiliary values C_x and S_w can also be set within reasonable ranges. A reference temperature compressibility correction of the appropriate parameters was used along with the adiabatic temperature-velocity relationship $T = T_{w,\mathrm{AD}} + (T_e - T_{w,\mathrm{AD}})U^2/U_e^2$. This is sufficiently accurate[13] for the present purpose of assessing the relative influence of injection, while simpler than using a Van Driest[14] transformation. The associated Mach number profile $M_o(y)$ needed in the SBLI solution is then calculated and the corresponding mass flow and momentum defect distributions $(1-\rho u/\rho_e U_e)$ and $(1-\rho u/\rho_e U_e)\, u/U_e$ integrated across the boundary layer to obtain the values of δ^*/δ and θ^*/δ, respectively, including the wall jet effect. The results shown in Fig. 3 illustrate how the mass and momentum addition from the wall jet substantially decreases δ^* and produces a fuller profile with a significantly reduced shape factor $H = \delta^*/\theta^*$ (although negative δ^* and θ^* are formally possible for large injection rates, we exclude such cases).

Implementation of these wall jet modifications is straightforward, except that the aforementioned modified integral properties must be fed back *after* the interactive pressure solution (which already includes the wall jet effect on the incoming boundary-layer profile) is carried out. Typical interactive pressure, displacement thickness, and skin-friction distributions so predicted are shown in Fig. 4. The increased boundary-layer profile fullness and smaller shape factor due to injection are seen to contract the streamwise extent of the interactive pressure rise significantly, as expected.[2,3] The corresponding displacement thickness growth angle $\mathrm{d}\Delta\delta^*/\mathrm{d}x$ also increases, whereas the overall increase $\Delta\delta^*/\delta_o^*$ is small because the net injection effect scales approximately with its influence on δ_o^*. The interactive skin-friction result (Fig. 4c) shows that the increased C_f level from the wall jet effect dominates both the fore and aft parts of the interaction zone, whereas in the vicinity of the shock foot the C_f reduction due to the steepened interactive pressure gradient caused by the injection dominates and the local value of $C_{f_{\min}}$ is actually reduced. Stated another way, the localized SBLI effect around the shock foot adversely counteracts the otherwise favorable C_f increase due to injection.

A comprehensive summary of the detailed parametric study[12] showing the up- and downstream influence distances (where the pressure rise is 5 and 95%, respectively, of the overall shock jump value) is presented in Fig. 5 as a function of both the magnitude and location of the jet velocity maximum for a typical supercritical flow. These results show that tangential injection can significantly reduce the overall streamwise interaction scale to a degree comparable to, or greater than, the unblown shape factor and/or Mach number effects.[12] The corresponding systematic influence of injection in increasing the streamwise slope of the interactive displacement thickness is shown in Fig. 6. This is important, since it

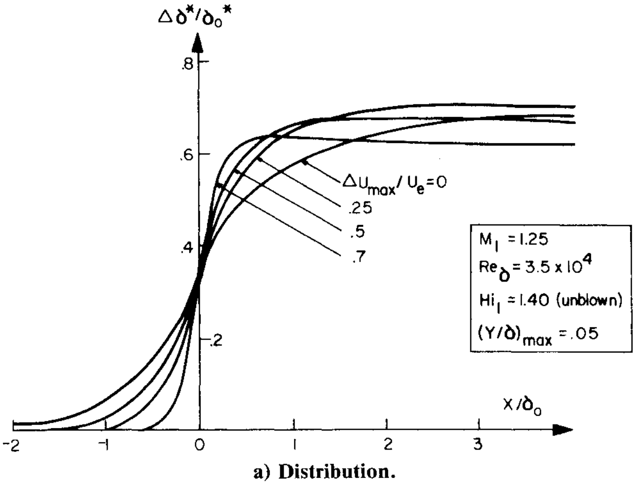


a) Upstream influence distance.

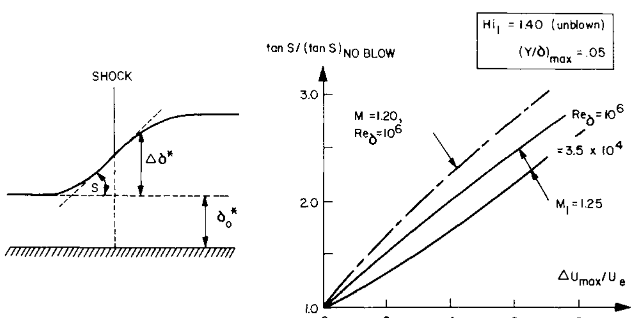


b) Downstream influence distance.

Fig. 5 Parametric study results: injection effect on the streamwise scale of the interaction.

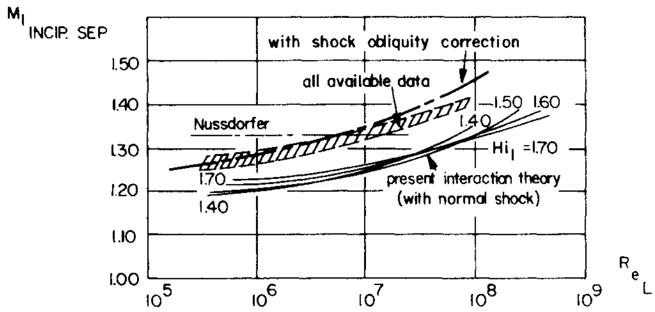


a) Distribution.

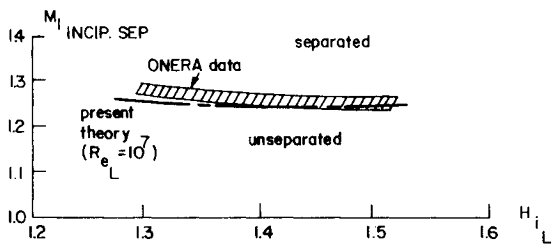


b) Viscous wedge angle.

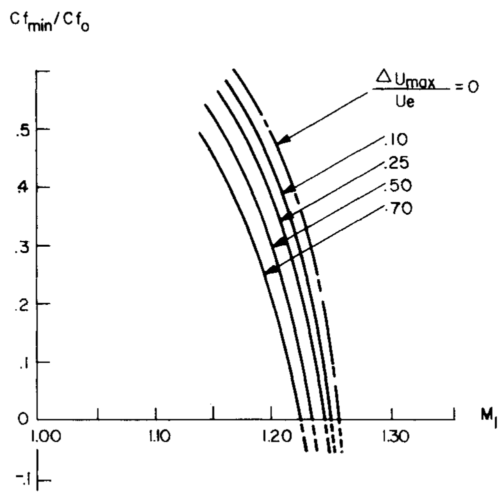
Fig. 6 Overall injection effects on the interactive displacement thickness.



a) Reynolds number effect.



b) Influence.



c) Injection effect.

Fig. 7 Incipient separation with upstream tangential injection.

relates to the effective "viscous wedge" angle sensed by the outer inviscid flow.[3,6,10]

The present theory, although it breaks down at separation, does yield a useful indication of incipient separation where $C_{f_{min}} \to 0$ owing to the detailed interactive skin-friction analysis[1] in the theory. The predicted results, including injection, are summarized in Fig. 7 in terms of the shock Mach number above which incipient separation occurs. Owing to the increased local interactive pressure gradient due to the shape factor reduction effect of injection, it is seen that $C_{f_{min}}$ is reduced in contrast to the corresponding overall increase in the up- and downstream C_f. Upstream tangential injection thus actually *hastens* the onset of local shock foot incipient separation for a given Reynolds number since it occurs at a slightly lower shock number as U_{max}/U_e is increased. This is, of course, in sharp contrast to the well-known[15] favorable separation-delaying effect of tangential injection observed for purely subsonic flows with a prescribed adverse pressure gradient and is due to the fact that the interactive pressure gradient enhancement effect of upstream injection is absent in the latter flows. There is presently no supporting data for this interesting result, however, since all available experimental studies have evidently been concerned exclusively with the regime of well-separated flow and the use of localized injection within the SBLI zone to suppress the separation.

Conclusions

The present theory is a useful tool in the analysis of the viscous-inviscid flowfield around supercritical aerodynamic bodies, providing a locally insertable interactive module astride the inviscid shock location driven by the attendant local boundary-layer properties, including an arbitrary non-equilibrium shape factor. As such, it facilitates study of employing tangential injection to modify the SBLI effect on the trailing-edge region flow of supercritical airfoils and enables proper treatment of shock/boundary-layer interaction effects in viscous-inviscid flowfield analysis programs for supercritical circulation-controlled wings.

Acknowledgment

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